Unit 1

Content Standards, Rationale, Strategies, Essential Questions

5.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.

Rationale

Students should be given ample opportunities to explore numerical expressions with mixed operations. They write expressions to express a calculation, e.g., writing 2 x (8 + 7) to express the calculation "add 8 and 7, then multiply by 2.", in other words, literacy in mathematics. Another example: Are students able to interpret and evaluate expressions, e.g., using their conceptual understanding of multiplication to interpret 3 x (18932 + 921) as being three times as large as 18932 +921, without having to calculate the indicated sum or product? This knowledge will prefigure later with variable expression such as three times an unknown length which is written 3 • *L (scaling)*. This work in grade five is viewed as exploratory rather than for attaining mastery so expressions should not be more complex than expressions they will use with the application of the associative or distributive property, e.g., (8 + 27) + 2 or (6 x 30) + (6 x 7).

Strategies

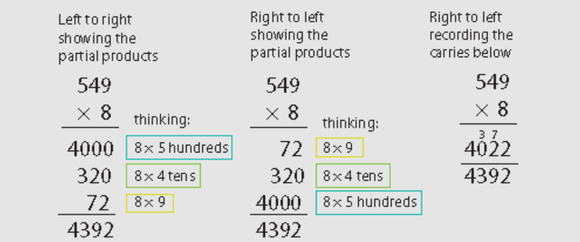
Begin with expressions that have two operations without any grouping symbols (multiplication or division combined with addition or subtraction) before introducing expressions with multiple operations. Using the same digits, with the operations in a different order, have students evaluate the expressions and discuss why the value of the expression is different. For example, have students evaluate 5 x 3 + 6 and 5 + 3 x 6. Discuss the conventions for parentheses and how and why they are used. Use activities like *How Many Dots* to have students understand why groups have to be computed first. They also need to interpret an expression without evaluating (solving like an equation).

5.NBT.5

Fluently multiply multi-digit whole numbers using the standard algorithm.

One component for understanding general methods for multiplication is to understand how to compute products of one-digit numbers and multiples of 10, 100, and 1000. Students can examine this structures by drawing area models or creating arrays with base ten blocks. Each part of the model must be connected to the numbers and symbols of each partial product. The goal is to explain how the structure insures these general methods will work every time.

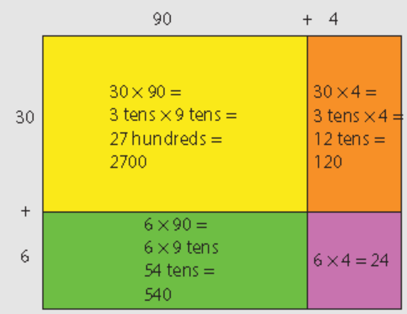
Computation of 8×549: Ways to record general methods



The first method proceeds from left to right, and the others from right to left. In the third method, the digits representing new units are written below the line rather than above 549, thus keeping the digits of the products close to each other, e.g., the 7 from $8 \times 9 = 72$ is written diagonally to the left of the 2 rather than above the 4 in 549.

Comparing the structures of general methods.

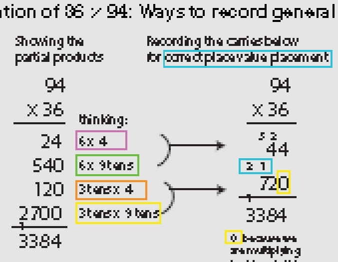
Computation of 36×94 connected with an area model



The products of like base-ten units are shown as parts of a rectangular region.

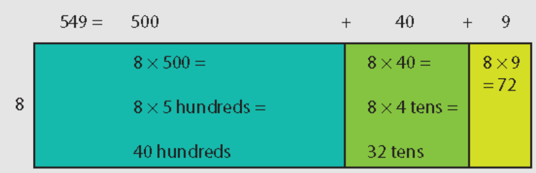
Students need to discuss and compare all methods

Computation of 36×94: Ways to record general methods



These proceed from right to left, but could go left to right. On the right, digits that represent newly composed tens and hundreds are written below the line instead of above 94. The digits 2 and 1 are surrounded by a blue box. The 1 from $30 \times 4 = 120$ is placed correctly in the hundreds place and the digit 2 from $30 \times 90 = 2700$ is placed correctly in the thousands place. If these digits had been placed above 94, they would be in incorrect places. Note that the 0 (surrounded by a yellow box) in the ones place of the second line of the method on the right is there because the whole line of digits is produced by multiplying by 30 (not 3).

Computation of 8×549 connected with an area model



Each part of the region above corresponds to one of the terms in the computation below.

$$\begin{aligned} 8 \times 549 &= 8 \times (500 + 40 + 9) \\ &= 8 \times 500 + 8 \times 40 + 8 \times 9. \end{aligned}$$

This can also viewed as finding how many objects are in 8 groups of 549 objects, by finding the cardinalities of 8 groups of 500, 8 groups of 40, and 8 groups of 9, then adding them.

$$\begin{aligned} 36 \times 94 &= (30 + 6) \times (90 + 4) \\ &= (30 + 6) \times 90 + (30 + 6) \times 4 \\ &= 30 \times 90 + 6 \times 90 + 30 \times 4 + 6 \times 4. \end{aligned}$$

The vertical method of recording is for computational purposes only.   Equations must written horizontally to be DEFINED as an equation. This is important.

TIP: *Students should be encouraged to estimate before solving. Number Talks in which students can only use mental math are ideal for practicing and understanding this skill.*

5.NBT.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Rationale
Division strategies in Grade 5 involve breaking the dividend apart into like base-ten units and applying the distributive property to find the quotient place by place, starting from the highest place. (Dividend can also be viewed as finding an unknown factor: the dividend is the product, the divisor is the known factor, and the quotient is the known factor, and the quotient is the unknown factor: the dividend is the product, the divisor is the known factor, and the quotient is the unknown factor.) Students continue to extend their fourth grade work on division, extending it to computation of whole number quotients with dividends of up to four digits and two-digit divisors. Estimation becomes relevant when extending to two-digit divisors. Understanding of rounding and the powers of ten will be applied. Even if students round appropriately, the resulting estimate may need to be adjusted. Partitive division and measurement division problems should be used to practice the skills. Remainders must be dealt with appropriately. For abstract practice numerous number talks again are a very effective strategy to practice reasoning and estimating division problems mentally. The focus should be on the structure – place value, powers of ten, multiplication and equal groups.

Recording division after an underestimate

1655 ÷ 27	$\left.\begin{array}{r} 1 \\ 10 \\ 50 \end{array}\right\} 61$
Rounding 27 to 30 produces the underestimate 50 at the first step but this method allows the division process to be continued	$\begin{array}{r} (30) \quad \\ 27\,\overline{)\,1655} \\ \underline{-1350} \\ 305 \\ \underline{-270} \\ 35 \\ \underline{-27} \\ 8 \end{array}$

Throughout this unit students will engage in a variety of non-routine multi-operation problem-types to develop and strengthen their:

- problem-solving skills
- understanding of how operations function and their relationships
- use of properties for ease of computing
- perseverance for solving problems
- ability to justify and examine other's strategies; explaining why a strategy works or doesn't work
- recording skills